Title: Playing with Primes

Brief Overview:

In these related activities, students will conduct an investigation of a prime-generation pattern working in pairs, and then apply some of the factoring concepts they have gained to factor monomials.

NCTM Content Standard/National Science Education Standard:

- Use factors, multiples, prime factorization, and relatively prime numbers to solve problems.

Grade/Level:

6 – 8/Middle school

Duration/Length:

Three 45-minute class periods

Student Outcomes:

Students will:

- Be more comfortable working with exponentiation.
- Be more familiar with the primes from 2 through 97.
- Be able to work with the PEMDAS strategy for order of operations.
- Be able to factor monomials using prime factorization, greatest common factor and/or repeated factors.

Materials and Resources:

- Scientific Calculator
- Response boards
- Resource: *Mathematics Magazine* (MAA) 2/93
- Worksheets
    - Tool #1: Powers of 2
    - Tool #2: Powers of 3
    - Tool #3: Primes from 2 through 97
    - Prime Power
    - All the Power to You!
    - Summative Assessment

Development/Procedures:

Lesson 1

Preassessment – Review several exponent examples, such as $2^4 = 16,\ 5^3 = 125,\ 6^2 = 36$. Emphasize how to find the answer using repeated factors prior to going to the calculator.
Review the order of operations. Consider using the pneumonic PEMDAS (please excuse my dear aunt Sally – parentheses, exponents, multiplication and division left to right, addition and subtraction left to right). Have the students apply the order of operations to show differences and similarities in the following:

$3 \times 4 - 2$
$3 \times (4 - 2)$ (Answers: 10, 6, 10)
$(3 \times 4) - 2$

Launch – Have the students complete "Tool #1: Powers of 2". Students will be finding answers from 2^0 to 2^{10} and may extend this list for homework. Remind them that $2^0 = 1$. One explanation can use the rule of exponents (done previously) that $\frac{2^4}{2^4} = 1$ because $\frac{16}{16} = 1$ and it equals 2^{4-4} which equals 2^0.

Teacher Facilitation – Presentation of the new concept. Work with the students to build their understanding of the concept. Get constant feedback through questioning and/or use of whiteboards to check that students are following the concept.

Student Application – Have the students complete "Tool #2: Powers of 3," following the same instructional procedures as above.

Construct Tool #3: Construct a list of primes from 2 through 97 using Erastosthenes' Sieve of Primes. Use the 100's chart, and start by having the students circle 2 and cross out all multiples of 2. Then circle 3 and cross out multiples of 3, etc. until only primes are left on their chart. Check the list as a class or by sharing with a partner. If computers are available, the sieve can be done at: www.faust.fr.bw.schule.de/mhb/eratosiv.htm or at: http://nlvm.usu.edu/en/NAV/frames_asid_158_g_3_t_1.html.

Embedded Assessment – Have students do several problems involving order of operations and exponents and display on whiteboards or use as an exit ticket.

Reteaching/Extension –

- For those who have not completely understood the lesson, review what is needed using an exit ticket.
- For those who have understood the lesson, take them to the next step in development of the concept.

Lesson 2

Preassessment – Use several examples from yesterday's lesson, especially those that the students had trouble with on their exit tickets.

Launch – Present the conjecture, "The difference in a power of 2 and a power of 3 in any order yield many prime numbers." Present the following :

$$5 = 9 - 4$$

$$5 = 3^2 - 2^2$$

So the prime 5 is equal to a power of 3 minus a power of 2.

Present more examples, and challenge the students to come up with more on their own.

$$73 = 81 - 8$$

$$73 = 3^4 - 2^3$$

$$3 = 4 - 1$$

$$3 = 2^2 - 3^0$$

$$101 = 128 - 27$$

$$101 = 2^7 - 3^3$$

Teacher Facilitation – After students have discovered several more examples, ask the main question of the investigation: "If the difference in a power of 2 and a power of 3 in any order yield prime numbers, is the converse of the statement true? Can any prime be written as a difference of a power of 2 and a power of 3? What is the LOWEST PRIME that proves the converse is false?" Refer to the worksheet, "Prime Power" to guide student work.

Student Application – Have the students work individually, in pairs or in groups, to find the lowest prime that does not satisfy the condition of being the difference between a power of 2 and 3.

Embedded Assessment – Evaluate the students' progress by reflecting: Did they find the number? How many numbers did they find that did work before they found one that didn't?

Reteaching/Extension – For homework, have them find another number that does not work or three primes higher than 97 that do work.

Lesson 3

Preassessment – Review law of division of exponential expressions and transition from numerical to variable expression.

$\frac{2^9}{2^7} = 2^{9-7} = 2^2$, then $\frac{x^9}{x^7} = x^2$

Launch – Work with students through the following example, $\frac{a^2}{a^6} = \frac{1}{a^4}$. Have the students simplify the expression using any one of the following methods. The students could divide the numerator and denominator by the GCF, a^2. They could also write each part out as repeated factors (2 factors of *a* in the numerator and 6 factors in the denominator. Divide out the common factors. They could use the formula that division of powers involves subtracting the exponents, as done in the preassessment. Assign the self-checking worksheet, "All the Power to You!". As the student finds the answer to each exercise, they find the letter associated with their answer and put it in the appropriate space in the answer section. If done correctly, they will get the message 'You are right'.

Teacher Facilitation – Students may work individually or in pairs to help each other through the worksheet.

Student Application – Students should check with a partner if having difficulties and help each other through the worksheet.

Embedded Assessment – The teacher should use either whiteboards or an exit ticket to assess whether the students understand the concepts.

Reteaching/Extension: If there are many problems with the worksheet and/or exit ticket, the teacher should review at the beginning of the next class.

Summative Assessment:

Assign the summative assessment at the end of this document.

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Tool # 1: Powers of 2 Name: _____

Simplify.

2^0	
2^1	
2^2	
2^3	
2^4	
2^5	
2^6	
2^7	
2^8	
2^9	
2^{10}	
2^{11}	
2^{12}	

Tool # 2: Powers of 3 Name: _____

Simplify.

3^0	
3^1	
3^2	
3^3	
3^4	
3^5	
3^6	
3^7	
3^8	
3^9	
3^{10}	
3^{11}	
3^{12}	

Tool #3: Primes from 2 through 97 Name: _____

Directions: Use the 100's chart below and the Sieve of Erastosthenes to generate a list of primes from 2 through 97.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Tool #3: Primes from 2 through 97 Name: __ANSWER KEY__

Directions: Use the 100's chart below and the Sieve of Erastosthenes to generate a list of primes from 2 through 97.

1	**2**	**3**	4	5	6	**7**	8	9	10
11	12	**13**	14	15	16	**17**	18	**19**	20
21	22	**23**	24	25	26	27	28	**29**	30
31	32	33	34	35	36	**37**	38	39	40
41	42	**43**	44	45	46	**47**	48	**49**	50
51	52	**53**	54	55	56	57	58	**59**	60
61	62	63	64	65	66	**67**	68	69	70
71	72	**73**	74	75	76	**77**	78	**79**	80
81	82	**83**	84	85	86	87	88	**89**	90
91	92	93	94	95	96	**97**	98	99	100

Prime Power Name: _____

The following primes are given in the form PRIME = Power of Two – Power of Three OR PRIME = Power of Three – Power of Two. Note that solutions are not unique, e.g. $5 = 3^2 - 2^2$ and $5 = 2^5 - 3^3$.

$2 = 3^1 - 2^0 = 3 - 1$

$3 = 2^2 - 3^0 = 4 - 1$

$5 = 3^2 - 2^2 = 9 - 4$

$7 = 2^4 - 3^2 = 16 - 9$

$11 = 3^3 - 2^4 = 27 - 16$

$13 = 2^8 - 3^5 = 256 - 243$

$17 = 3^4 - 2^6 = 81 - 64$

$19 = 3^3 - 2^3 = 27 - 8$

$23 = 3^3 - 2^2 = 27 - 4$

$29 = 2^5 - 3^1 = 32 - 3$

$31 = 2^5 - 3^0 = 32 - 1$

$37 = 2^6 - 3^3 = 64 - 27$

$\rightarrow \rightarrow \rightarrow$41 cannot be done!$\leftarrow \leftarrow \leftarrow$

Extension/ homework:

$47 = 2^7 - 3^4 = 128 - 81$

$61 = 2^6 - 3^1 = 64 - 3$

$73 = 3^4 - 2^3 = 81 - 8$

$79 = 3^4 - 2^1 = 81 - 2$

$101 = 2^7 - 3^3 = 128 - 27$

$127 = 2^7 - 3^0 = 128 - 1$

$139 = 3^7 - 2^{11} = 2187 - 2048$

$179 = 3^5 - 2^6 = 243 - 64$

$509 = 2^9 - 3^1 = 512 - 3$

$1021 = 2^{10} - 3^1 = 1024 - 3$

All the Power to You! Name: _____

Directions: Find the answer- find the letter that corresponds to your answer, then place that letter under the appropriate space below:
Solve and check with your partner:

1. $\frac{a^8}{a^{10}}$ 2. $\frac{2a^5}{3a^4}$ 3. $\frac{6x^3}{8x^5}$

4. $\frac{36a^2x^5}{36a^4x^5}$ 5. $\frac{3b^3}{6b^2}$ 6. $\frac{32ax^3}{48a^3x^2}$

7. $\frac{3gh^3m}{5g^2h^2m}$ 8. $\frac{60x^2y^2z^3}{90x^5yz}$ 9. $\frac{8g^2x}{18g^3x^2}$

10. $\frac{23m^4y^5}{7m^6y^3}$ 11. $\frac{3a^3y^2}{27ay^2}$

3	6	8	10	1	11	4	9	5	7	2

A. $\frac{23y^2}{7m^2}$ B. $\frac{13y^2}{23b}$ C. $\frac{8}{18gx}$ D. $\frac{3}{2b}$ E. $\frac{a^2}{9}$ F. $\frac{3hm}{5g^2}$ G. $\frac{b}{2}$ H. $\frac{3h}{5g}$

I. $\frac{4}{9gx}$ J. $\frac{8gx}{18}$ K. $\frac{a}{b}$ L. $\frac{2m}{3n}$ M. $\frac{4a}{6}$ N. $\frac{4}{6a}$ O. $\frac{2x}{3a^2}$ P. $\frac{6a}{9}$

Q. 12 R. $\frac{1}{a^2}$ S. $\frac{2y}{3x^2}$ T. $\frac{2a}{3}$ U. $\frac{2yz^2}{3x^3}$ V. $\frac{6}{8x}$ W. $\frac{2h}{g}$ X. $\frac{4x}{2a}$

Y. $\frac{3}{4x^2}$ Z. $\frac{1}{a^4}$

All the Power to You! Name: __ANSWER KEY_____

Directions: Find the answer- find the letter that corresponds to your answer, then place that letter under the appropriate space below:
Solve and check with your partner:

1. $\frac{a^8}{a^{10}} = \frac{1}{a^2}$

2. $\frac{2a^5}{3a^4} = \frac{2a}{3}$

3. $\frac{6x^3}{8x^5} = \frac{3}{4x^2}$

4. $\frac{36a^2x^5}{36a^4x^5} = \frac{1}{a^2}$

5. $\frac{3b^3}{6b^2} = \frac{b}{2}$

6. $\frac{32ax^3}{48a^3x^2} = \frac{2x}{3a^2}$

7. $\frac{3gh^3m}{5g^2h^2m} = \frac{3h}{2g}$

8. $\frac{60x^2y^2z^3}{90x^5yz} = \frac{2yz^2}{3x^3}$

9. $\frac{8g^2x}{18g^3x^2} = \frac{4}{9gx}$

10. $\frac{23m^4y^5}{7m^6y^3} = \frac{23y^2}{7m^2}$

11. $\frac{3a^3y^2}{27ay^2} = \frac{a^2}{9}$

3	6	8	10	1	11	4	9	5	7	2
Y	O	U	A	R	E	R	I	G	H	T

A. $\frac{23y^2}{7m^2}$ B. $\frac{13y^2}{23b}$ C. $\frac{8}{18gx}$ D. $\frac{3}{2b}$ E. $\frac{a^2}{9}$ F. $\frac{3hm}{5g^2}$ G. $\frac{b}{2}$ H. $\frac{3h}{5g}$

I. $\frac{4}{9gx}$ J. $\frac{8gx}{18}$ K. $\frac{a}{b}$ L. $\frac{2m}{3n}$ M. $\frac{4a}{6}$ N. $\frac{4}{6a}$ O. $\frac{2x}{3a^2}$ P. $\frac{6a}{9}$

Q. 12 R. $\frac{1}{a^2}$ S. $\frac{2y}{3x^2}$ T. $\frac{2a}{3}$ U. $\frac{2yz^2}{3x^3}$ V. $\frac{6}{8x}$ W. $\frac{2h}{g}$ X. $\frac{4x}{2a}$

Y. $\frac{3}{4x^2}$ Z. $\frac{1}{a^4}$

Summative Assessment Name: _____

For exercises 1 – 3, simplify each expression.

1. 5^3

2. $3\left(2^2 - 1\right)$

3. $\dfrac{6x^3}{9x^2}$

4. Answer the following.
   - What is the value of 2^5?
   - What is the value of 5^2?
   - Does $2^5 = 5^2$? Use mathematics to explain your answer. Use words, symbols, or both in your explanation.

Summative Assessment Name: ___ANSWER KEY__

For exercises 1 – 3, simplify each expression.

1. $5^3 = 125$

2. $3\left(2^2 - 1\right) = 9$

3. $\dfrac{6x^3}{9x^2} = \dfrac{2x}{3}$

4. Answer the following.
   - What is the value of 2^5?
     32
   - What is the value of 5^2?
     25
   - Does $2^5 = 5^2$? Use mathematics to explain your answer. Use words, symbols, or both in your explanation.
     No, you are using a different base taken to the power in each problem – the base must be the repeated factor that you multiply the number of times stated in the exponent.